SOLUTION OF INVERSE ACOUSTIC SCATTERING PROBLEMS USING SHAPE SENSITIVITY ANALYSIS

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Several problems in science and engineering can be posed as optimization problems with constraints given by boundary-value problems. For example, finding the shape of an object immersed in an acoustic medium given scattering information at certain locations and frequencies is one of them. In this inverse acoustic scattering problem the cost functional is the mismatch between the scattering pattern obtained from a trial solution and the measured one. The scattering pattern corresponding to the trial solution satisfies the wave-equation with the Sommerfeld radiation condition imposed at infinity and appropriate boundary conditions imposed at the surface of the scatterer. This exterior acoustic boundary-value problem is a constraint imposed on the admissible scattering patterns. The "variable" to be optimized is the shape of the scatterer, or equivalently, the domain where the boundary-value problem is posed.

On the other hand, it is fair to say that mathematical programming, or the solution of nonlinear constrained optimization problems in several variables, has reached maturity, and there are several robust algorithms that can be used to treat different classes of problems. If the functionals and constraints in the optimization problem are differentiable, it makes sense to use derivative information to efficiently find the optimum of the mathematical program.

Therefore, assuming that the problem mentioned above is differentiable, we ask the following questions: (1) is it possible to recast this problem as a constrained optimization problem in several variables and therefore use gradient-based optimization algorithms to solve it?; (2) if so, what are the corresponding optimization parameters?; and (3) how do we calculate the derivatives in the most efficient way?

The answer to these questions is the subject of the present work, and more specifically, its application to the solution of inverse acoustic scattering problems.